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A Mathematical Representation of an Advanced Helicopter for Piloted Simulator Investigations of Control- System and Display Variations

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SYMBOLS

A_{1s}, B_{1s}	lateral and longitudinal swashplate angle, respectively, deg
a	blade slope-of-the-lift curve, per rad
$a_{x_{cg}}, a_{y_{cg}}$	acceleration along body X and Y axis, respectively, as measured at aircraft c.g., ft/sec ²
C_p, C_T	power and thrust coefficients, respectively
g	acceleration due to gravity, 32.2 ft/sec ²
h	altitude, ft
\dot{h}	altitude rate, ft/sec
$I_{()}$	moment of inertia about body ()-axis, slug-ft ²
I_{xz}	product of inertia in body axes, slug-ft ²
$K_{()}$	linear gain
$L_{()}$	body axis rolling moment derivative
$M_{()}$	body axis pitching moment derivative = $\frac{1}{I_{yy}} \frac{\partial M}{\partial ()}$, rad/sec ² /()
m	mass, slugs
$N_{()}$	body axis yawing moment derivative = $\frac{1}{I_{zz}} \frac{\partial N}{\partial ()}$, rad/sec ² /()
p, q, r	body axis roll, pitch, yaw rates. respectively, rad/sec
s	Laplace operator $\sigma \pm j\omega$
u_a, v_a, w_a	airspeed components along body X, Y, and Z axes, respectively, ft/sec, knots
V	total airspeed, ft/sec, knots
$X_{()}, Y_{()}, Z_{()}$	body axis longitudinal, lateral and vertical force derivatives, respectively = $\frac{1}{m} \frac{\partial X, Y, \text{ or } Z}{\partial ()}$, ft/sec ² /()
\hat{x}, \hat{y}	estimates of inertial longitudinal and lateral velocity, respectively, ft/sec
\ddot{x}, \ddot{y}	longitudinal and lateral inertial accelerations, ft/sec ²

λ	total inflow ratio
λ_0	λ in hover
$\delta_{B_1}, \delta_{A_1},$ $\delta_{TR}, \delta_{\theta_0}$	SCAS input to longitudinal cyclic pitch, lateral cyclic pitch, tail rotor collective pitch, and main rotor collective pitch, respectively, deg
$\delta_e, \delta_a,$ δ_r, δ_c	pilot's controller positions - longitudinal cyclic, lateral cyclic, pedal, and collective, respectively, in.
θ_0, θ_{TR}	main rotor and tail rotor collective pitch, respectively, deg
μ	advance ratio
σ	solidity ratio
ϕ, θ, ψ	Euler roll, pitch, and yaw angles, respectively, rad

A MATHEMATICAL REPRESENTATION OF AN ADVANCED HELICOPTER FOR
PILOTED SIMULATOR INVESTIGATIONS OF CONTROL SYSTEM
AND DISPLAY VARIATIONS

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SUMMARY

This report documents a mathematical model of an advanced helicopter; the model is suitable for use in control/display research involving piloted simulation. The general design approach for the six-degree-of-freedom equations of motion is to use the full set of nonlinear gravitational and inertial terms of the equations and to express the aerodynamic forces and moments as the reference values and first-order terms of a Taylor series expansion about a reference trajectory defined as a function of longitudinal airspeed. Provisions for several different specific and generic flight control systems are included in the model. The logic required to drive various flight control and weapon delivery symbols on a pilot's electronic display is also provided. Finally, the model includes a simplified representation of low-altitude wind and turbulence effects. This model has been used in a piloted simulator investigation, recently conducted at Ames Research Center, of the effects of control-system and display variations for an attack helicopter mission.

1. INTRODUCTION

To provide a real-time simulation for research programs that involve wide variations in helicopter flight control system characteristics and electronic display format and logic, it is necessary to produce a model of the helicopter that requires a minimum of digital computer capacity yet is sufficiently realistic for the task being simulated. One candidate technique has been applied to two fixed-base simulations for control/display research involving VTOL aircraft in terminal-area operations (refs. 1, 2); this approach includes the full nonlinear set of kinematic terms in six-degree-of-freedom equations of motion, but uses linearized aerodynamic terms assuming small perturbations about reference conditions that vary as functions of a single independent variable. As described herein, this particular technique was applied to derive a mathematical model of an advanced helicopter valid for small perturbations from level flight conditions for the airspeed range of -40 to +160 knots. Sufficient flexibility is retained in the model to facilitate the conversion to different basic helicopter aerodynamic characteristics.

The derivation of the expressions for the aerodynamic forces and moments is presented in section 2 of this report.

A separate segment of the model is used to generate the control surface positions as functions of cockpit control positions and elements of the aircraft state vector. A total of six different flight control systems is provided. Their characteristics are described in section 3.

Another portion of the model contains the logic that converts the elements of the aircraft state vector into the parameters required to drive the symbols on the pilot's electronic display. These parameters -- which convey aircraft orientation, situation, command, and fire control information to the pilot -- are defined in section 4.

Section 5 describes the model of the low-altitude wind and turbulence implemented to provide external disturbances to the systems under investigation. The model includes steady wind, wind shear in magnitude and direction, and turbulence components whose characteristics vary with mean wind speed and altitude. Conclusions are presented in section 6.

These elements of the mathematical model of an advanced helicopter have been used in a piloted moving-base simulator study of the effects on flying qualities of variations in both flight control system and display characteristics for an attack helicopter mission (ref. 3).

2. AERODYNAMICS

General

The total aerodynamic forces and moments required for the six-degree-of-freedom equations of motion are generated as the summation of reference and first-order terms of a Taylor series expansion about a reference trajectory defined as a function of longitudinal airspeed (u_a). Function generation system subroutines (ref. 4) are utilized to produce the values for the following parameters as functions of the single variable u_a :

1. Reference values for total forces and moments -- X_R , Y_R , Z_R , and M_R
2. Reference values for aircraft motion and control variables -- w_R , $A1_{s_R}$, $B1_{s_R}$, θ_{o_R} , and θ_{TR_R}
3. Values for the aircraft stability and control parameters -- e.g., X_w and Z_{θ_0}

The reference values for the total forces and moments are specified at 20-knot intervals of the independent variable for $-40 \text{ knots} \leq u_a \leq +160 \text{ knots}$. Each of the remaining dependent variables is specified at 40-knot intervals of the independent variable. Linear interpolation is used to determine the value of each parameter between these breakpoints.

Approach

As an example of the technique used to calculate the total aerodynamic forces and moments, the X-force equation is expressed as:

$$\frac{X_a}{m} = X_R(u_a) + X_w(u_a)\Delta w_a + X_q(u_a)q + X_{B_{1s}}(u_a)\Delta B_{1s} + X_{\theta_o}(u_a)\Delta\theta_o$$

where

$$\Delta w_a = w_a - w_{aR}(u_a)$$

$$\Delta B_{1s} = B_{1s} - B_{1sR}(u_a)$$

$$\Delta\theta_o = \theta_o - \theta_{oR}(u_a)$$

Preliminary computer-generated values of the stability and control derivatives and reference values of the aircraft motion and control variables have been provided by Hughes Helicopters as a function of flight condition; values derived from the flight-test data presented in reference 5 supplement these computer-generated data where possible.

The reference value of the X-force (X_R) is calculated by taking the partial derivative of X_a with respect to u_a :

$$\frac{1}{m} \frac{\partial X_a}{\partial u_a} \equiv X_u = \frac{dX_R}{du_a} - X_w \frac{dw_R}{du_a} - X_{B_{1s}} \frac{dB_{1sR}}{du_a} - X_{\theta_o} \frac{d\theta_{oR}}{du_a}$$

Transposing,

$$\frac{dX_R}{du_a} = X_u + X_w \frac{dw_R}{du_a} + X_{B_{1s}} \frac{dB_{1sR}}{du_a} + X_{\theta_o} \frac{d\theta_{oR}}{du_a}$$

Therefore, using the dummy variable σ :

$$X_R(u_a) = X_R(u_a = 0) + \int_0^{u_a} \frac{dX_R}{d\sigma} \cdot d\sigma$$

where $X_R(u_a = 0) = g \sin \theta_H$ and θ_H = hover trim pitch attitude.

For a given airspeed, the accuracy of the linear approximations of the total aerodynamic forces and moments is dependent in part on the magnitude of the perturbations from the reference trajectory. No analysis was conducted to investigate the errors that result from neglecting the nonlinear effects caused by large perturbations. Therefore, to improve confidence in the validity of experimental results, the task to be performed should in general involve only

"small" perturbations from the selected reference trajectory; this is particularly true when the model is used for piloted simulation.

Ground Effect

To account for the effect of the terrain on the aircraft dynamics, the stability parameter Z_h is included in the model. The derivation of the value of this parameter proceeds as follows. From momentum theory:

$$\frac{2C_T}{a\sigma} = \frac{\theta_o}{3} \left(1 + \frac{3}{2} \mu^2 \right) + \frac{\lambda}{2}$$

For $\mu = 0$:

$$\theta_o = 3 \left(\frac{2C_T}{a\sigma} - \frac{\lambda}{2} \right)$$

For the advanced helicopter,

$$C_T = 0.0066$$

$$a = 5.75$$

$$\sigma = 0.092$$

From reference 6, the value of λ in ground effect is expressed as:

$$\lambda = \lambda_o \left[1 + (G - 1) \left(\frac{V - 30}{30} \right)^2 \right]$$

For $V = 0$,

$$\lambda = G\lambda_o$$

where

$$\lambda_o = -\sqrt{\frac{C_T}{2}} = -0.0575$$

$$G = 0.25 + \frac{h_{cg}}{48}$$

Therefore,

$$\theta_o = 0.09653 + 0.001797 h_{cg}, \text{ rad}$$

and

$$\frac{\partial \theta_o}{\partial h} = 0.001797, \text{ rad/ft}$$

Therefore, for $V = 0$:

$$Z_h = -Z_{\theta_0} \frac{\partial \theta_0}{\partial h} = (4.562)(0.103) = 0.47 \text{ ft/sec}^2/\text{ft}$$

According to reference 6, this effect diminishes to zero as airspeed approaches 30 ft/sec and is only apparent for altitudes less than 1 rotor diameter. In the model, this effect is only present for altitudes less than 50 ft AGL. (Because no direct measurement of height above the simulated terrain is available, altitude AGL is estimated by subtracting a constant value representing mean terrain altitude from the actual altitude above the terrain board reference.) The parameter Z_h decreases linearly with velocity, reaching a value of zero for $u_a = -40$ and $+40$ knots.

Coupling Derivatives

The longitudinal and lateral-directional aerodynamics of the basic model are uncoupled with the exception of yawing moment due to tail rotor collective pitch inputs. An option which adds perturbations to the basic aerodynamic forces and moments to account for coupling effects is available. The following coupling effects are included: (1) longitudinal equations, v_a , p , r , A_{1S} , θ_{TR} ; and (2) lateral-directional equations, w_a , q , θ_0 , B_{1S} .

Summary of Equations

Perturbation variables-

$$DWB = WB - WBR$$

$$DTHET\emptyset = THET\emptyset - THET\emptyset R$$

$$DA1S = A1S - A1SR$$

$$DB1S = B1S - B1SR$$

$$DTHETTR = THETTR - THETTRR$$

where WBR , $THET\emptyset R$, $A1SR$, $B1SR$, and $THETTRR$ are all generated by function generator system subroutines as functions of UB .

X-force equation-

$$FAX = XMASS\{XQ*QB + XW*DWB + XB1S*DB1S + XTH\emptyset*DTHET\emptyset + XREF\}$$

where XQ , XW , $B1S$, $XTH\emptyset$, and $XREF$ are all generated as functions of UB .

Y-force equation-

$$FAY = XMASS*\{YP*PB + YR*RB + YV*VB + YAlS*DAIS + YTHTR*DTHETTR + YREF\}$$

where YP, YR, YV, YAlS, YTHTR, and YREF are all generated as functions of UB.

Z-force equation-

$$FAZ = XMASS*\{ZQ*QB + ZW*DWB + ZBlS*DBIS + ZTH\emptyset*DTHET\emptyset + ZH*DH + ZREF\}$$

where ZQ, ZW, ZBlS, ZTH \emptyset , ZH, and ZREF are all generated as functions of UB and

$$DH = \begin{cases} HAGL - 50 & \text{for } HAGL \leq 50 \text{ ft} \\ 0 & \text{for } HAGL > 50 \text{ ft} \end{cases}$$

where HAGL = HCG - HTER

L-moment equation-

$$TAL = XIXX*\{ULP*PB + ULR*RB + ULV*VB + ULAlS*DAIS + ULTTR*DTHETTR\}$$

where ULP, ULR, ULV, ULAlS, and ULTTR are all generated as functions of UB.

M-moment equation-

$$TAM = XIYY*\{UMQ*QB + UMW*DWB + UMBIS*DBIS + UMTH\emptyset*DTHET\emptyset + UMREF\}$$

where UMQ, UMW, UMBIS, UMTH \emptyset , and UMREF are all generated as functions of UB.

N-moment equation-

$$TAN = XIZZ*\{UNP*PB + UNR*RB + UNV*VB + UNTH\emptyset*DTHET\emptyset \\ + UNTTR*DTHETTR + UNAlS*DAIS\}$$

where UNP, UNR, UNV, UNTH \emptyset , UNTTR, and UNAlS are all generated as functions of UB.

The values of the reference forces and moments, stability and control parameters, and reference aircraft motion and control variables are presented in the appendix as functions of u_a at the designated breakpoints.

The optional perturbations to the basic expressions for total aerodynamic forces and moments to account for coupling effects are:

$$DELFAZ = XMASS*\{UKPBX*PB + UKRBX*RB + UKVBX*VB + UKDAISX*DAIS \\ + UKDTHTRX*DTHETTR\}$$

$$\text{DELFAY} = \text{XMASS} * \{ \text{UKQBY} * \text{QB} + \text{UKDWBY} * \text{DWB} + \text{UKDB1SY} * \text{DB1S} + \text{UKDTH0Y} * \text{DTHET0} \}$$

$$\begin{aligned} \text{DELFAZ} = \text{XMASS} * \{ \text{UKPBZ} * \text{PB} + \text{UKRBZ} * \text{RB} + \text{UKVBZ} * \text{VB} + \text{UKDA1SZ} * \text{DA1S} \\ + \text{UKDTHTRZ} * \text{DTHETTR} \} \end{aligned}$$

$$\text{DELTAL} = \text{XIXX} * \{ \text{UKQBL} * \text{QB} + \text{UKDWBL} * \text{DWB} + \text{UKDB1SL} * \text{DB1S} + \text{UKDTH0L} * \text{DTHET0} \}$$

$$\begin{aligned} \text{DELTAM} = \text{XIYY} * \{ \text{UKPBM} * \text{PB} + \text{UKRBM} * \text{RB} + \text{UKVBM} * \text{VB} + \text{UKDA1SM} * \text{DA1S} \\ + \text{UKDTHTRM} * \text{DTHETTR} \} \end{aligned}$$

$$\text{DELTAN} = \text{XIZZ} * \{ \text{UKQBN} * \text{QB} + \text{UKDWBN} * \text{DWB} + \text{UKDB1SN} * \text{DB1S} + \text{UKDTH0N} * \text{DTHET0} \}$$

The values for the coupling derivatives are presented in the appendix.

3. FLIGHT CONTROL SYSTEMS

General

Six control system configurations are provided:

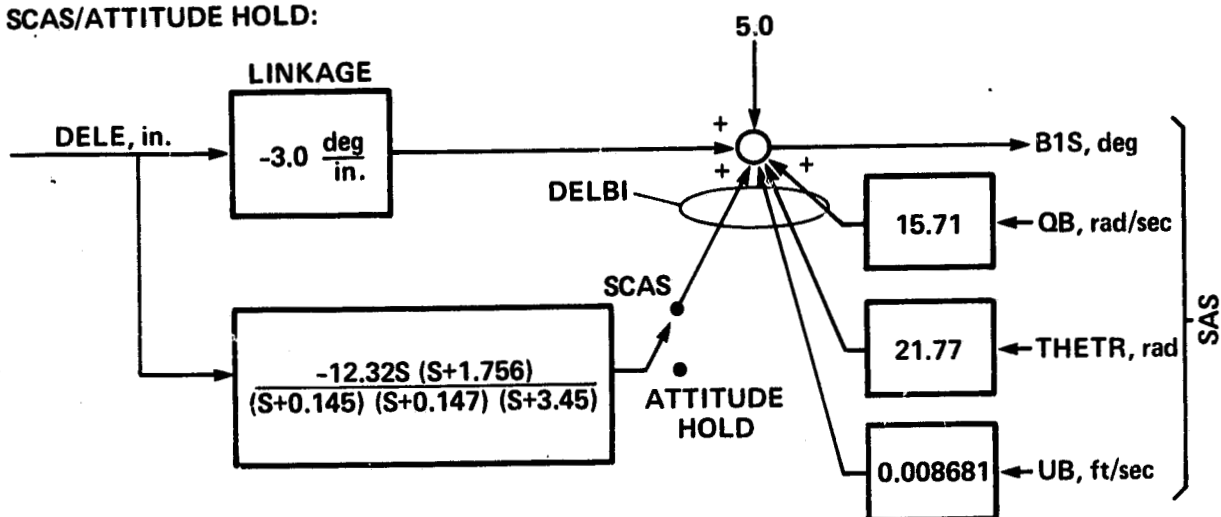
1. Mechanical -- pitch, roll, yaw SCAS OFF
2. SCAS ON -- pitch, roll, yaw; individually or collectively
3. Attitude Hold -- pitch, roll, yaw
4. Hover augmentation -- pitch and roll
5. Heading hold -- yaw
6. Vertical augmentation -- collective

Configurations (1) through (3) are specific control system configurations based in part on information derived from references 5 and 7 and on unpublished stability and control data. Configurations (4) through (6) are generic control systems judged to represent useful control system variations for an experimental investigation based on the attack helicopter mission.

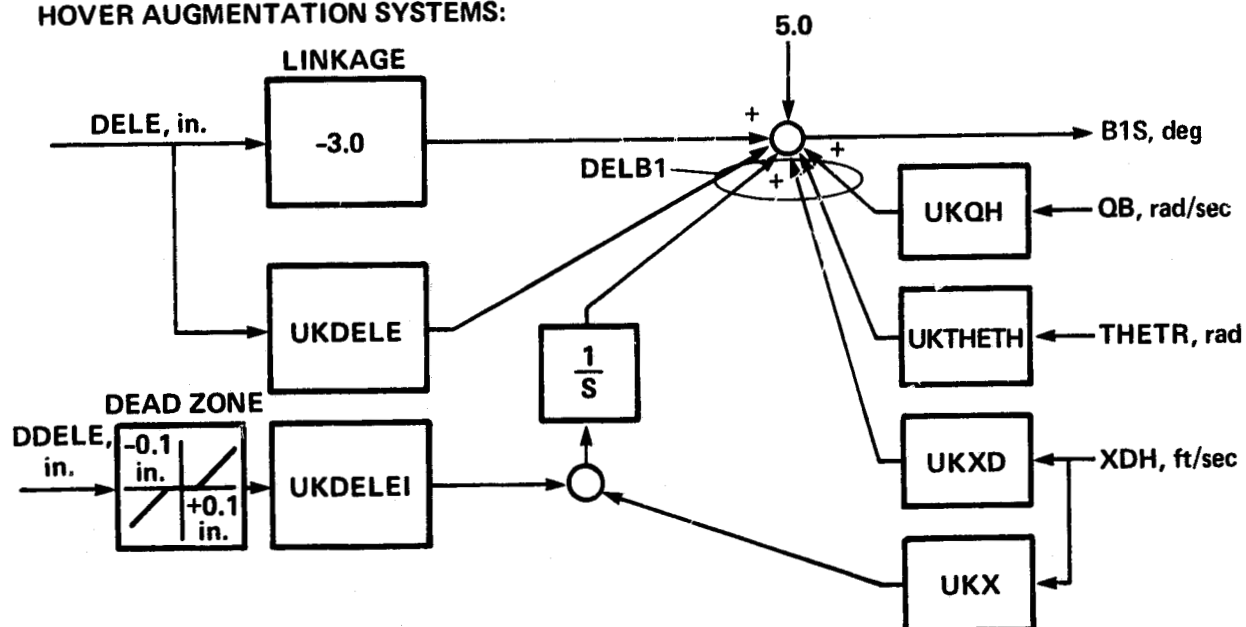
In general, a digital representation of the control system transfer functions is obtained by the use of the z-transform; using programs described in reference 4, the appropriate difference equations are obtained from the corresponding s-plane transfer functions.

Block diagrams of the various control system configurations are presented in figure 1.

SCAS/ATTITUDE HOLD:

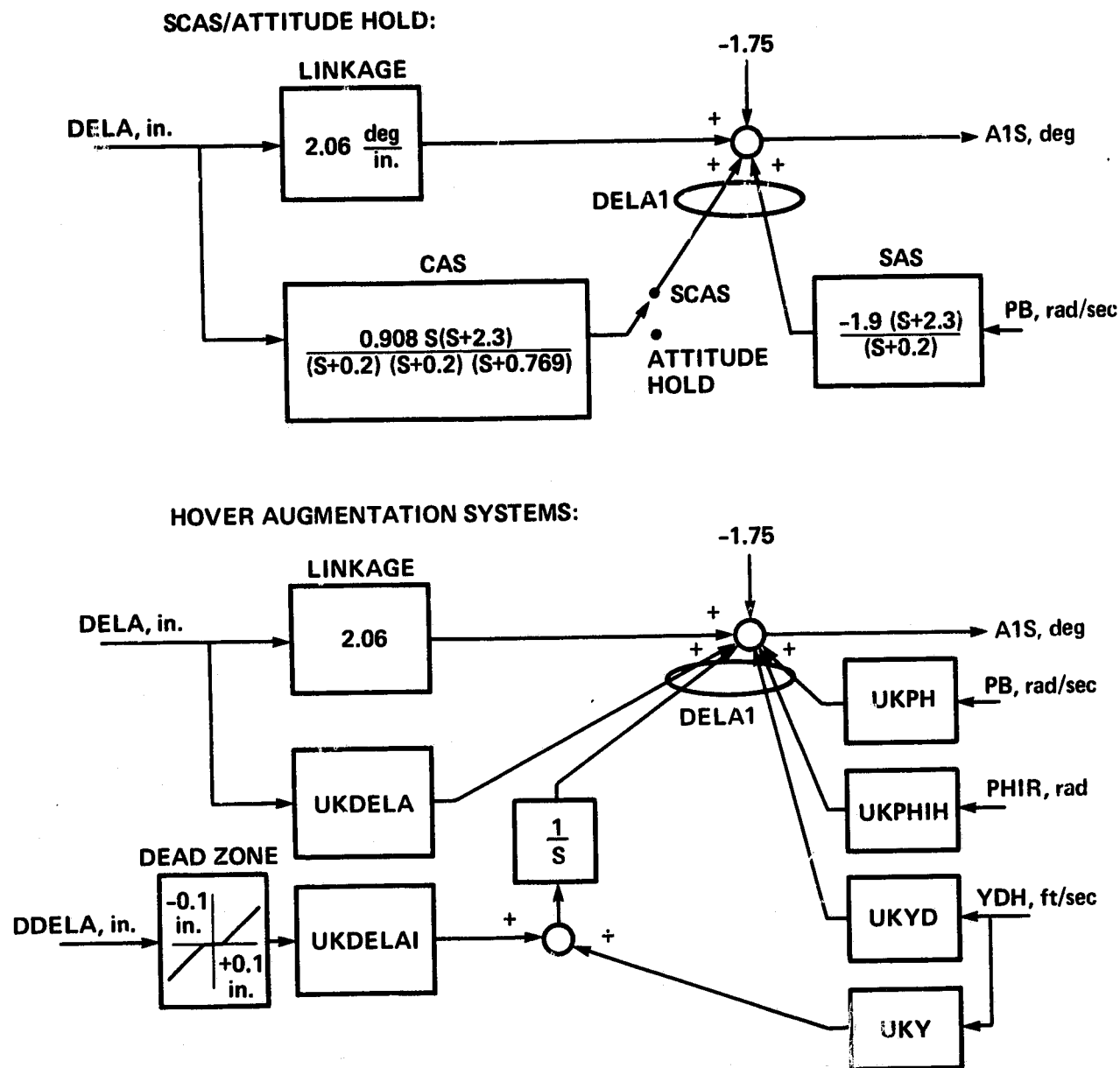


HOVER AUGMENTATION SYSTEMS:



(a) Pitch.

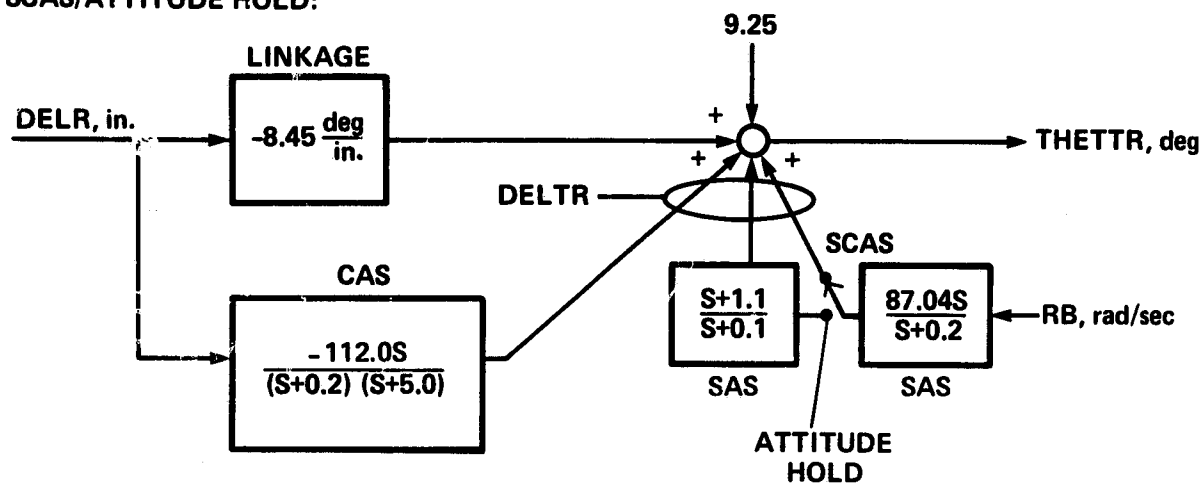
Figure 1.- Advanced helicopter control systems.



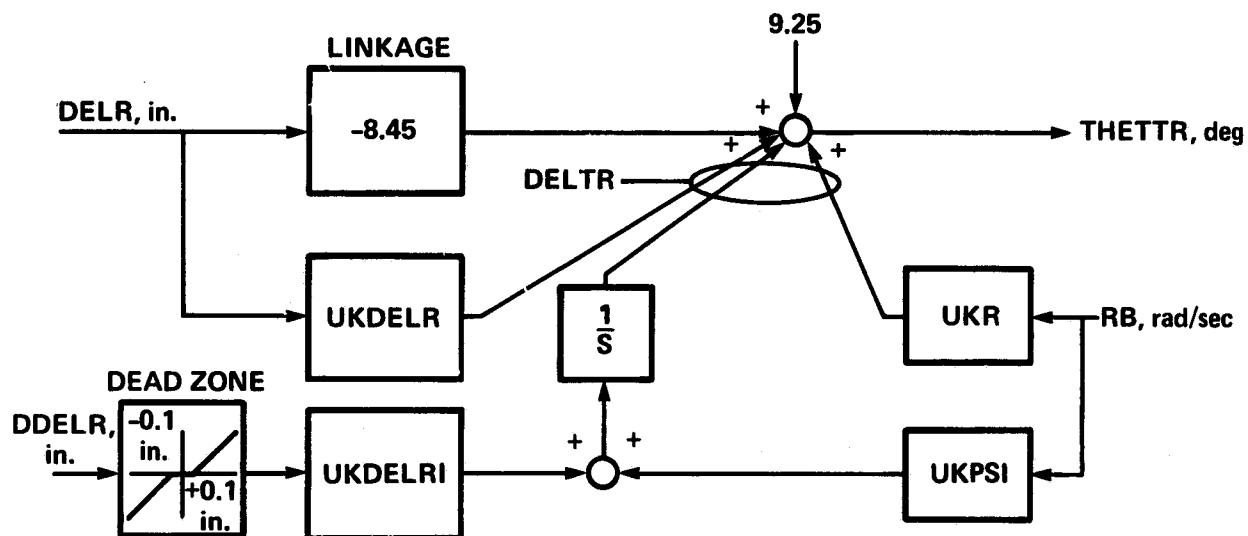
(b) Roll.

Figure 1.- Continued.

SCAS/ATTITUDE HOLD:



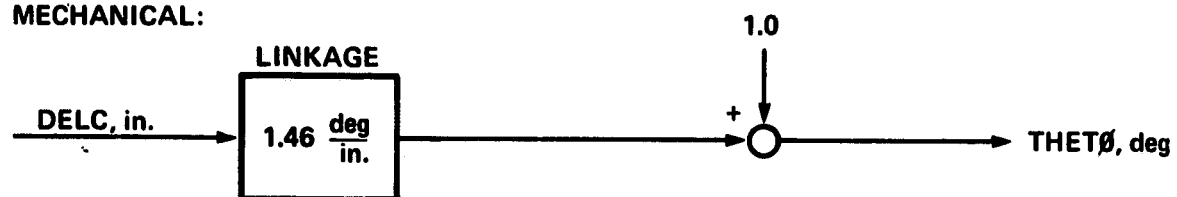
HOVER AUGMENTATION SYSTEMS:



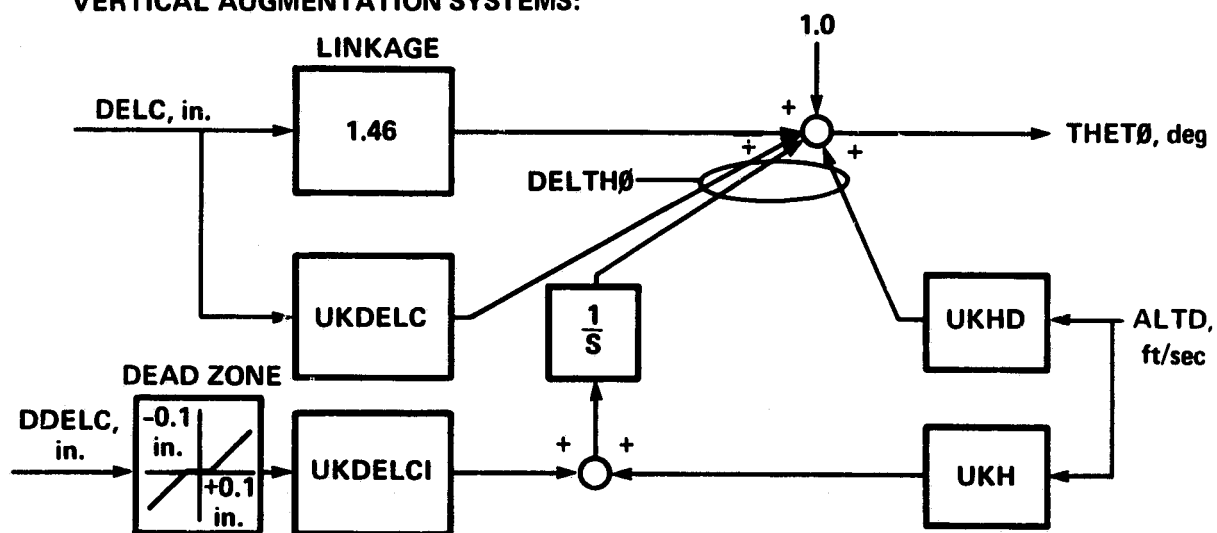
(c) Yaw - $V < 50$ knots.

Figure 1.- Continued.

MECHANICAL:



VERTICAL AUGMENTATION SYSTEMS:



(d) Collective.

Figure 1.- Concluded.

To provide the proper control force characteristics, the simulator cockpit is equipped with a control loader system described in reference 4. The reference 3 experiment used the following control force and displacement characteristics, derived from reference 7, to simulate the advanced helicopter.

	Controller			
	Lateral	Longitudinal	Directional	Collective
Force gradient, lb/in.	0.3	0.5	7.0	Adjustable friction
Breakout force, lb	1.3	1.0	7.0	Adjustable friction
Travel, in.	±4.5	±5.0	±2.75	0-12

Mechanical Flight Controls

The baseline mechanical flight control system uses pilot inputs of (1) longitudinal cyclic δ_e , (2) lateral cyclic δ_a , (3) directional controls δ_r , and (4) collective δ_c to determine, respectively, (1) longitudinal swash-plate angle B_{1S} , (2) lateral swash-plate angle A_{1S} , (3) tail rotor collective pitch θ_{TR} , and (4) main rotor collective pitch, θ_o . The relationships between the pilot control position and control surface position, derived from reference 5, are as follows.

Longitudinal-

$$B_{1S} = 5.0 - 3.0 \delta_e \quad \text{Limits} \begin{cases} \delta_e: \pm 5 \text{ in.} \\ B_{1S}: +20^\circ, -10^\circ \end{cases}$$

Lateral-

$$A_{1S} = -1.75 + 2.06 \delta_a \quad \text{Limits} \begin{cases} \delta_a: \pm 4.5 \text{ in.} \\ A_{1S}: +7.5^\circ, -11^\circ \end{cases}$$

Directional-

$$\theta_{TR} = 9.25 - 8.45 \delta_r \quad \text{Limits} \begin{cases} \delta_r: \pm 2.75 \text{ in.} \\ \theta_{TR}: +32.5^\circ, -14^\circ \end{cases}$$

Collective-

$$\theta_o = 1.0 + 1.46 \delta_c \quad \text{Limits} \begin{cases} \delta_c: 0-12 \text{ in.} \\ \theta_o: 1-18.5^\circ \end{cases}$$

Stability and Control Augmentation System (SCAS)

Limited authority SCAS actuators produce additional control surface motion in response to sensed aircraft motion parameters (SAS) and pilot control inputs (CAS) in the longitudinal, lateral, and directional axes. The SCAS control mode may be selected by the researcher for each axis individually or for all three axes collectively. The transfer functions for the SCAS are presented below together with the simplifications employed for the purposes of the simulation.

Longitudinal SCAS-

$$\delta B_1 = \frac{\delta B_1}{\theta} \cdot \theta + \frac{\delta B_1}{u} \cdot u + \frac{\delta B_1}{\delta e} \cdot \delta e$$

where

$$\frac{\delta B_1}{\theta} (s) = \frac{8.54 s^2 (s + 1.756)}{(s + 0.1)(s + 0.145)} + \frac{10.62(s + 0.3)(s + 0.975)}{(s + 0.15)} \sim \text{deg/rad}$$

Simplifying,

$$\begin{aligned} \frac{\delta B_1}{\theta} (s) &\approx \frac{8.54 s^2 (s + 1.756) + 10.62(s + 0.1)(s + 0.3)(s + 0.975)}{(s + 0.1)(s + 0.15)} \\ &= \frac{19.16(s^3 + 1.545 s^2 + 0.2327 s + 0.01621)}{(s + 0.1)(s + 0.15)} \\ &= \frac{19.16(s + 1.386)[s^2 + 2(0.72)(0.11)s + (0.11)^2]}{(s + 0.1)(s + 0.15)} \end{aligned}$$

$$\frac{\delta B_1}{\theta} (s) \approx 15.71(s + 1.386)$$

and

$$\frac{\delta B_1}{u} (s) = \frac{4.452 \times 10^{-3}(s + 0.3)(s + 0.975)}{(s + 0.15)(s + 1.0)} \sim \text{deg/ft/sec}$$

Simplifying,

$$\frac{\delta B_1}{u} (s) \approx 8.681 \times 10^{-3}$$

Finally,

$$\frac{\delta B_1}{\delta e} (s) = \frac{-12.32 s(s + 1.756)}{(s + 0.145)(s + 0.147)(s + 3.45)}$$

Lateral SCAS-

$$\delta A_1 = \frac{\delta A_1}{\phi} \cdot \phi + \frac{\delta A_1}{\delta a} \cdot \delta a$$

where

$$\begin{aligned} \frac{\delta A_1}{\phi} (s) &= \frac{-1.461 s^2 (s + 2.3)}{(s + 0.1)(s + 0.2)} - \frac{1.45 s (s + 2.28)}{(s + 0.87)} \sim \text{deg/rad} \\ &= \frac{-2.911 s (s + 2.3) (s + 0.0175) (s + 0.5686)}{(s + 0.1)(s + 0.2)(s + 0.87)} \end{aligned}$$

Simplifying,

$$\frac{\delta A_1}{\phi} (s) \approx -1.90 \frac{s(s + 2.3)}{(s + 0.2)}$$

Finally,

$$\frac{\delta A_1}{\delta a} (s) = \frac{0.908 s(s + 2.3)}{(s + 0.2)(s + 0.2)(s + 0.769)} \sim \text{deg/in.}$$

Directional SCAS-

$$\delta_{TR} = \frac{\delta_{TR}}{r} \cdot r + \frac{\delta_{TR}}{\delta_r} \cdot \delta_r + \frac{\delta_{TR}}{\phi} \cdot \phi + \frac{\delta_{TR}}{v} \cdot v$$

where

$$\frac{\delta_{TR}}{r} (s) = 87.04 \left(\frac{s}{s + 0.2} \right) \sim \text{deg/rad/sec}$$

and

$$\frac{\delta_{TR}}{\delta_r} (s) = \frac{-112 s}{(s + 0.2)(s + 5.0)} \sim \text{deg/in.}$$

For $V \leq 50$ knots, $\delta_{TR}/\phi = \delta_{TR}/v = 0$. For $V > 50$ knots only,

$$\frac{\delta_{TR}}{\phi} (s) = \frac{-324.3 K s^2}{(s + 0.2)(s + 10)} - \frac{614.8 K s}{(s + 0.2)(s + 10)} \sim \text{deg/rad}$$

where $K = 0.5 - 0.00333(V - 50)$ ($V \sim$ knots). Simplifying,

$$\frac{\delta_{TR}}{\phi} (s) \approx \frac{-32.43 K s (s + 1.896)}{(s + 0.2)}$$

and

$$\frac{\delta_{TR}}{V} (s) = \frac{-831.4}{V(s + 14.7)} \sim \text{deg/ft/sec}$$

$$\frac{\delta_{TR}}{V} (s) \approx -\frac{57.3}{V}$$

SCAS limits- SCAS actuator authority limits are presented in reference 7 as percentages of equivalent full controller deflection as follows:

1. 10% for pitch and roll SCAS
2. 20% for yaw SCAS

The following control surface limits result:

$$\delta B_1 \rightarrow \pm 3.0^\circ$$

$$\delta A_1 \rightarrow \pm 1.86^\circ$$

$$\delta_{TR} \rightarrow \pm 9.3^\circ$$

Attitude Hold

According to reference 7, the attitude hold mode is available below $V = 50$ knots by switching out the CAS in the pitch and roll axes, that is

$$\delta B_1 = \frac{\delta B_1}{\theta} \cdot \theta + \frac{\delta B_1}{u} \cdot u$$

and

$$\delta A_1 = \frac{\delta A_1}{\phi} \cdot \phi$$

and by providing a pseudo-heading-hold feature in yaw, that is

$$\delta_{TR} = 87.04 \left(\frac{s}{s + 0.2} \right) \left(\frac{s + 1.1}{s + 0.2} \right) r \sim \text{deg}$$

Hover Augmentation

The implementation of a hover position hold system through the pitch and roll SCAS actuators consists of the following logic:

$$\delta B_1 = K_{\delta_e} \delta_e + K_{f\delta_e} \int \delta_e + K_q q + K_\theta \theta + K_{\dot{x}_h} \dot{x}_h + K_x \epsilon_{x_h}$$

and

$$\delta A_1 = K_{\delta_a} \delta_a + K_{f\delta_a} \int \delta_a + K_p p + K_\phi \phi + K_{\dot{y}_h} \dot{y}_h + K_y \epsilon_{y_h}$$

where the h subscript indicates positions and inertial velocities in an aircraft heading-referenced axis system with origin at the nominal center of gravity, and the ϵ terms indicate position errors from the pilot-designated hover point.

The standard simulation software described in reference 4 calculates the north and east components of the aircraft inertial velocity (VN and VE, respectively). The transformation from these earth-referenced velocity components to the heading-referenced components (XDH and YDH) utilizes the sine and cosine of the heading angle (SPSI and CPSI) as follows:

$$XDH = VN * CPSI + VE * SPSI$$

$$YDH = -VN * SPSI + VE * CPSI$$

The heading-referenced position errors ϵ_{x_h} and ϵ_{y_h} are calculated through an integration of the appropriate velocity components which commences when the pilot designates a hover point.

These heading-referenced quantities are also used by the display dynamics program to calculate the positions of various symbols on the pilot's electronic display.

Heading Hold

With the heading hold mode selected, the directional axis SCAS equation becomes:

$$\delta_{TR} = K_{\delta_r} \delta_r + K_{f\delta_r} \int \delta_r + K_r r + K_\psi \epsilon_\psi$$

The intent of this control mode is to provide a yaw rate command-heading hold control system through the pilot's directional controls.

Vertical Augmentation

With vertical augmentation selected, a simulated collective SCAS is implemented, consisting of the following logic:

$$\delta_{\theta_0} = K_{\delta_c} \delta_c + K_{f\delta_c} \int \delta_c + K_{\dot{h}} \dot{h} + K_h \epsilon_h$$

The objective of this SCAS mode is to provide an altitude rate command-altitude hold control system through the pilot's collective stick.

Summary of Equations

In general, the various control systems to be investigated are implemented as perturbations on the basic mechanical flight control system; that is:

$$A1S = -1.75 + 2.06*DELA + DELA1$$

with A1S limited to +7.5° to -11°, DELA limited to ±4.5 in., and DELA1 limited to ±1.86°.

$$B1S = 5.0 - 3.0*DELE + DELB1$$

with B1S limited to +20° to -10°, DELE limited to ±5 in., and DELB1 limited to ±3.0°.

$$THETTR = 9.25 - 8.45*DELR + DELTR$$

with THETTR limited to +32.5° to -14°, DELR limited to ±2.75 in., and DELTR limited to ±9.3°.

$$THET\emptyset = 1.0 + 1.46*DELC + DELTH\emptyset$$

with THET \emptyset limited to 1.0° to 18.5°, and DELC limited to 0.0 to 12.0 in.

The perturbation quantities DELA1, DELB1, DELTR, and DELTH \emptyset are calculated using logic determined by the control mode selected (tables 1 through 4). (The SCAS actuator limits specified above are nominal values and may be set to any other values by the researcher.)

TABLE 1.- δ_{B1} LOGIC

Control mode	DELB1 =
Pitch SCAS OFF	---
Pitch SCAS ON	$\left[\frac{-12.32 s(s + 1.756)}{(s + 0.145)(s + 0.147)(s + 3.45)} \right] *DELE + 21.77*THETR$ $+ 15.71*QB + 0.008681*UB$
Attitude hold	21.77*THETR + 15.71*QB + 0.008681*UB
Hover hold	$UKDELE*DELE + \frac{1}{s} *(UKDELEI*DDELE + UKX*XDH) + UKTHETH*THETR$ $+ UKQH*QB + UKXD*XDH$ <p>where DDELE is the perturbation of DELE from its value at the time of engagement passed through a dead zone of ±0.1 in.</p>

TABLE 2.- δ_{A_1} LOGIC

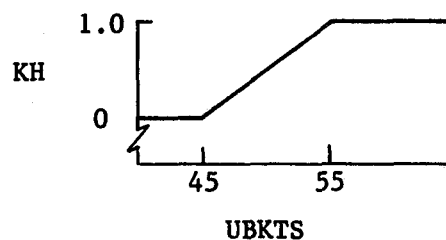
Control mode	DELA1 =
Roll SCAS OFF	---
Roll SCAS ON	$\left[\frac{0.908 s(s + 2.3)}{(s + 0.2)(s + 0.2)(s + 0.769)} \right] *DELA - 1.9 * \left(\frac{s + 2.3}{s + 0.2} \right) *PB$
Attitude hold	$-1.9 * \left(\frac{s + 2.3}{s + 0.2} \right) *PB$
Hover hold	$UKDELA *DELA + \frac{1}{s} * (UKDELA1 *DDELA + UKY *YDH) + UKPHIH *PHIR$ $+ UKPH *PB + UKYD *YDH$

where DDELA is the perturbation of DELA from its value at the time of engagement passed through a dead zone of ± 0.1 in.

TABLE 3.- $\delta_{\theta_{TR}}$ LOGIC

Control mode	DELTR =
Yaw SCAS OFF	---
Yaw SCAS ON	$\left[\frac{-112.0 s}{(s + 0.2)(s + 5.0)} \right] *DELTR + 87.04 * \left(\frac{s}{s + 0.2} \right) *RB$ $+ KH * \left[(-21.61 + 0.06391 *UB) * \left(\frac{s + 1.896}{s + 0.2} \right) *PB - \frac{57.3}{UB} *VB \right]$

where



Attitude hold	$\left[\frac{-112.0 s}{(s + 0.2)(s + 5.0)} \right] *DELTR + 87.04 * \left(\frac{s}{s + 0.2} \right) \left(\frac{s + 1.1}{s + 0.1} \right) *RB$
Heading hold	$UKDELTR *DELTR + \frac{1}{s} * (UKDELRI *DDELTR + UKPSI *RB) + UKRH *RB$

where DDELTR is the perturbation of DELTR from its value at the time of engagement passed through a dead zone of ± 0.1 in.

TABLE 4.- δ_{θ_0} LOGIC

Control mode	DELTH θ =
Collective SCAS OFF	---
Altitude hold	$UKDEL C * DELC + \frac{1}{s} * (UKDEL CI * DDEL C + UKAH * ALTD) + UKHD * ALTD$ where DDEL C is the perturbation of DELC from its value at the time of engagement passed through a dead zone of ± 0.1 in.

4. DISPLAY DYNAMICS

General

The purpose of the display dynamics portion of the mathematical model is to produce the signals used to drive the moving symbols on the various candidate electronic display formats. These signals are either simply elements of the aircraft state vector or the result of certain logic applied to selected state vector elements to produce the desired dynamic characteristics. The moving symbols are organized in this section on the basis of the type of information they convey; that is, orientation, situation, command, and fire control.

An additional function of this portion of the program is to alter the display logic as a function of four discrete display modes — cruise, transition, hover, and bob-up — which are selected manually by the pilot. Reference 9 describes the operational requirements associated with each display mode as:

1. Cruise — high-speed level flight en route to the forward edge of the battle area
2. Transition — low-speed nap-of-the-earth maneuvers, such as dash, quick stop, and sideward flight
3. Hover — stable hover with minimum drift
4. Bob-Up — unmask and remask maneuvers over a selected horizontal ground position

In addition to the electronic display symbol drive logic, the display dynamics program also provides signals for the following cockpit instruments:

1. Attitude-director indicator (ADI)
2. Horizontal situation indicator (HSI)
3. Radar altimeter

4. Barometric altimeter
5. Instantaneous vertical speed indicator (IVSI)
6. Normal accelerometer
7. Airspeed indicator
8. Engine torque

Flight Control Display Logic

A typical electronic display format is illustrated in figures 2 through 5. The primary symbols used by the pilot to control his aircraft are the velocity vector, cyclic director symbol, and hover position symbol (fig. 3). The logic and scaling of the parameters that drive these symbols vary as a function of display mode.

Cruise mode- These symbols do not appear in the cruise display mode.

Transition mode- The velocity vector is driven directly by the horizontal components of Doppler velocity in the transition mode; that is, displayed vertical motions of the vector are driven by the longitudinal component of heading-referenced velocity (XDH) while its lateral motions are driven by YDH.

The displayed vertical motion of the cyclic director symbol with respect to the tip of the velocity vector is driven by washed-out pitch attitude with a washout time constant of 50 sec. Laterally, the symbol is driven by roll attitude for roll angles greater than 5.73° and by washed-out roll attitude for smaller values of roll angle. For the latter case, the washout time constant is 10 sec.

Hover mode- For the smaller values of velocity encountered in the hover, the velocity vector is driven by a complementary-filtered estimate of the longitudinal and lateral heading-referenced inertial velocities, \hat{x} and \hat{y} , respectively.

These estimates are obtained by blending low-frequency Doppler velocity data with high-frequency inertial acceleration data. Assuming 1-g flight and small values of pitch and roll attitudes, the inertial accelerations may be expressed as:

$$\ddot{x} \approx a_{x_{cg}} - g\theta$$

$$\ddot{y} \approx a_{y_{cg}} + g\phi$$

The velocity estimates are expressed as:

$$\hat{x} = \left(\frac{1}{T_x s + 1} \right) \dot{x} + \left[\frac{s}{(s + \lambda_x)(s + 1/T_x)} \right] (a_{x_{cg}} - g\theta)$$

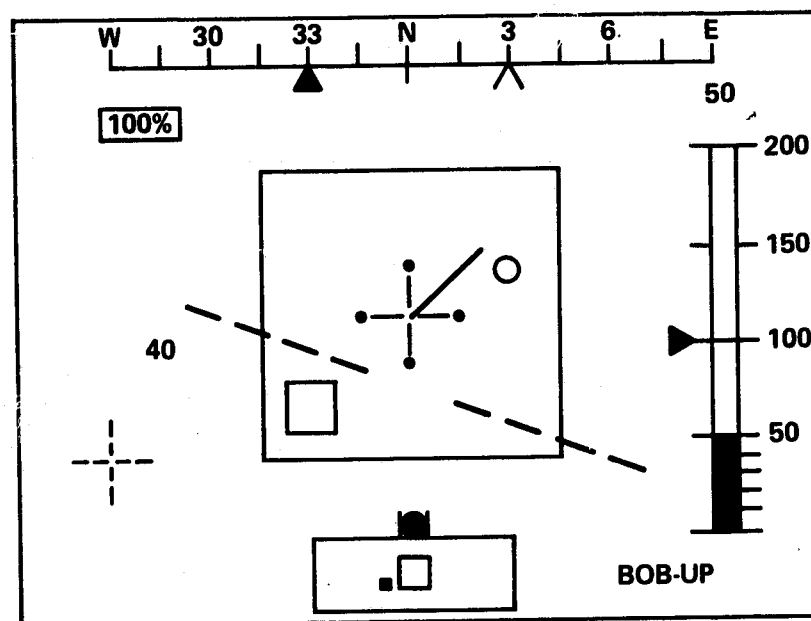


Figure 2.- Electronic display format (ref. 8).

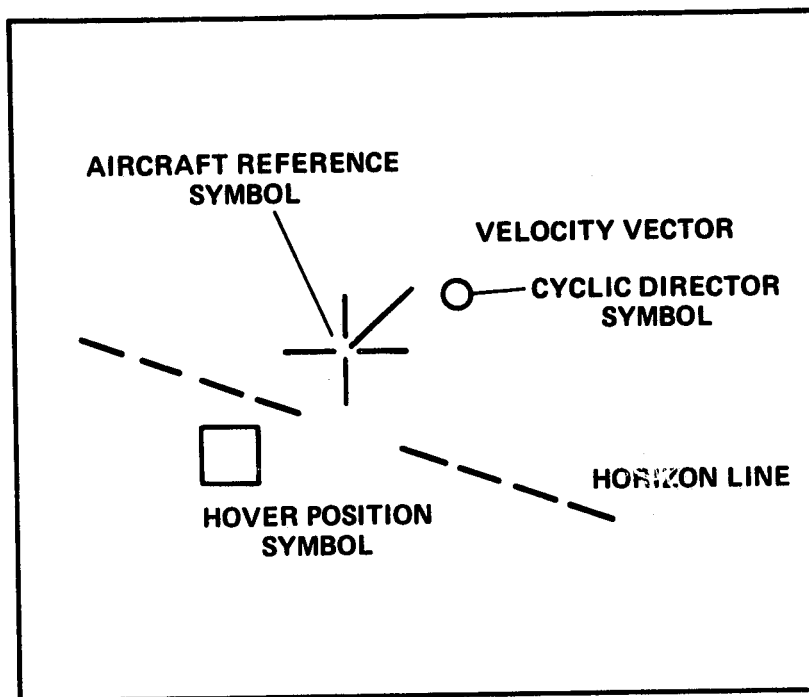


Figure 3.- Central symbology.

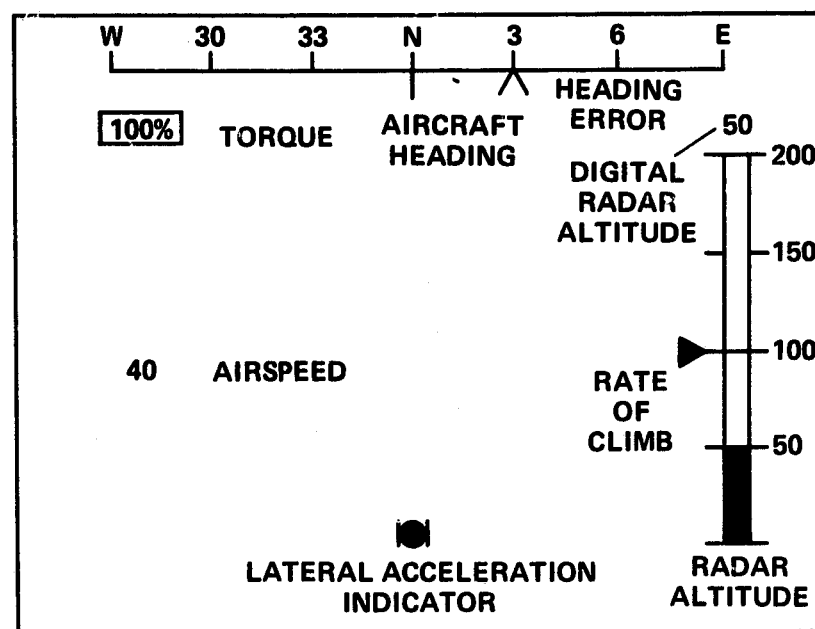


Figure 4.- Peripheral symbology.

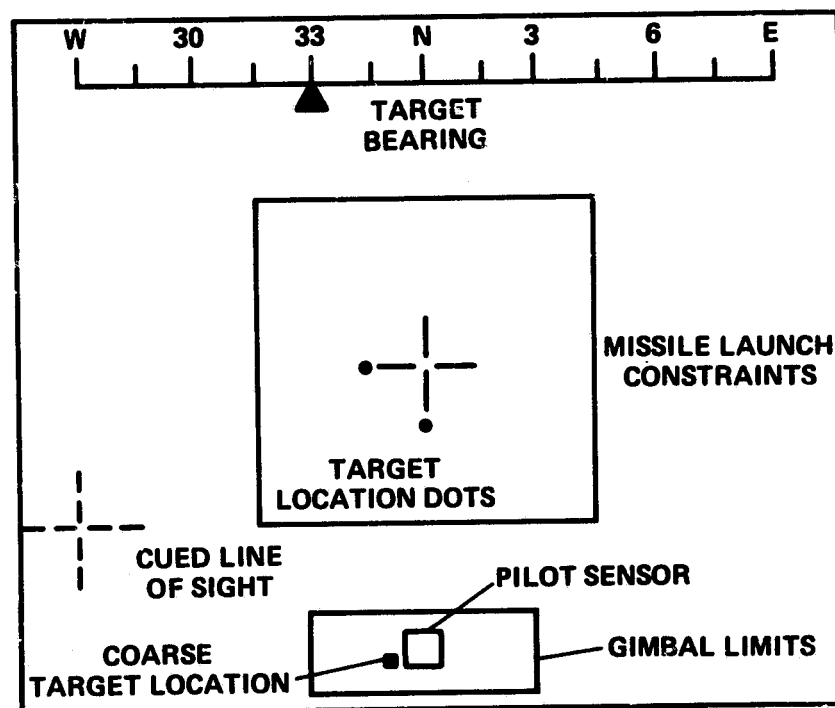


Figure 5.- Fire control symbology.

$$\hat{\dot{y}} = \left(\frac{1}{T_y s + 1} \right) \dot{y} + \left\{ \frac{s}{(s + \lambda_y)[s + (1/T_y)]} \right\} (a_{y_{cg}} + g\phi)$$

The filter time constants are retained as variables. A further simplification suggested in reference 9 is to assume that both $a_{x_{cg}}$ and $a_{y_{cg}}$ are negligible with respect to the gravitational acceleration components. Both the complete and the simplified expressions for \hat{x} and \hat{y} are implemented in the program.

The cyclic director symbol is driven by washed-out pitch attitude (10-sec time constant) and washed-out roll attitude (10-sec time constant).

These changes in logic occur instantaneously at the time of the switch from transition to hover mode.

Bob-up mode- The logic driving the velocity vector and cyclic director symbol remains the same as the hover mode logic. The hover position symbol is now driven vertically by EXH and laterally by EYH where EXH and EYH are the integrals of XDH and YDH, respectively, with integration commencing at the time the bob-up display mode is selected. Finally, a command heading symbol, which has remained fixed on the display, is now driven by the difference between the current heading and the heading that existed at the time the bob-up display mode was selected.

Torque Equation

One of the status parameters displayed to the pilot is engine torque in percent (fig. 4). Since no engine or rotor dynamics are considered in the model, an approximate relationship between engine torque and main rotor collective pitch is used to drive this symbol. This relationship is derived as follows. The relationship between power required and power turbine torque may be expressed as:

$$P = N_2 \frac{2\pi}{60} Q$$

where

P = shaft power = power required, (ft-lb)/sec

N_2 = power turbine speed = 2×10^4 rpm

Q = power turbine torque, ft-lb

In hover, the total power coefficient is equal to the sum of the induced, profile, and miscellaneous power coefficients:

$$C_p = C_{p_i} + C_{p_o} + C_{p_{misc}}$$

Typically,

$$C_p \doteq 1.67 C_{p_1}$$

where

$$C_{p_1} = \left(\frac{C_T^3}{2} \right)^{1/2}$$

For the advanced helicopter in hover ($C_T = 0.0066$)

$$C_p \doteq 6.35 \times 10^{-4}$$

$$P = 1.05 \times 10^5 \text{ (ft-lb)/sec}$$

$$HP = \frac{P}{550} = 1900$$

This value of hover horsepower required is approximately 62% of the available 3,086 hp in the advanced helicopter. Therefore the torque required to hover out of ground effect is assumed to be 62%. To determine the relationship between a perturbation from the trim value of main rotor collective pitch and the resultant perturbation in torque from the nominal hover value, the following technique is used. Since $C_p = C_{p_1} + C_{p_0} + C_{p_{misc}}$,

$$\frac{\partial C_p}{\partial \theta_0} = \frac{\partial C_{p_1}}{\partial C_T} \cdot \frac{\partial C_T}{\partial \theta_0}$$

and

$$\frac{\partial P}{\partial \theta_0} = \frac{\partial C_{p_1}}{\partial C_T} \cdot \frac{\partial T}{\partial \theta_0} \cdot \Omega R$$

Therefore,

$$\frac{\partial Q}{\partial \theta_0} = \frac{\partial P}{\partial \theta_0} \cdot \frac{60}{2\pi} N_2 = \frac{\partial C_{p_1}}{\partial C_T} \cdot \frac{\partial T}{\partial \theta_0} \cdot \Omega R \cdot \frac{60}{2\pi} N_2$$

but

$$\frac{\partial C_{p_1}}{\partial C_T} = \frac{3}{2} \left(\frac{C_T}{2} \right)^{1/2}$$

and

$$\frac{\partial T}{\partial \theta_0} \doteq -mZ_{\theta_0}$$

Therefore,

$$\frac{\partial Q}{\partial \theta_0} \doteq \left[\frac{3}{2} \left(\frac{C_T}{2} \right)^{1/2} \right] (-mZ_{\theta_0}) (\Omega R) \frac{60}{2\pi} N_2$$

For the advanced helicopter in hover out of ground effect:

$$C_T = 0.0066$$

$$m = 453.42 \text{ slugs}$$

$$Z_{\theta_0} = -4.562 \text{ ft/sec}^2/\text{deg}$$

$$\Omega R = (30.26 \text{ rad/sec})(24 \text{ ft})$$

$$N_2 = 2 \times 10^4 \text{ rpm}$$

Therefore,

$$\frac{\partial Q}{\partial \theta_0} \doteq 61.86 \text{ (ft-lb)/deg}$$

The maximum torque in foot-pounds is estimated by dividing the hover value of torque in foot-pounds by that value in percent; that is,

$$Q_{\max} \doteq \frac{499.7}{0.6157} = 811.6 \text{ ft-lb}$$

Therefore,

$$\frac{\partial Q}{\partial \theta_0} \doteq \frac{61.86}{811.6} \cdot 100 = 7.6\%/\text{deg}$$

This hover approximation for torque is also used for forward flight since the experiment for which the model was designed (ref. 3) only involved hover and low-speed flight.

Fire Control Display Logic

Figure 5 illustrates typical symbology used to assist the pilot in his weapon delivery task. The logic driving the cued line of sight symbol causes the symbol to overlay a simulated fixed ground target in the background image. The background imagery simulates the video received from a sensor, such as forward-looking infrared or low-light-level television mounted at some location on the aircraft. This location is variable to allow the investigation of the effects caused by the sensor offset from the pilot's eye position.

The derivation of the logic for the cued line of sight symbol proceeds as follows. Let (x_T, y_T, z_T) represent the target position in an aircraft body axis system with origin at the sensor location. Therefore the desired values of displayed target azimuth and elevation are:

$$A_z = \tan^{-1} \frac{y_T}{x_T}$$

$$E_\ell = -\sin^{-1} \frac{z_T}{R_s}$$

where $R_s = \sqrt{x_T^2 + y_T^2 + z_T^2}$. The values of x_T , y_T , and z_T are calculated as follows. Target position in an earth-referenced coordinate system with origin at the sensor location is expressed as:

$$x_{TP} = R_i \cos(A_{zi} + \psi_i) - \Delta_x$$

$$y_{TP} = R_i \sin(A_{zi} + \psi_i) - \Delta_y$$

$$z_{TP} = z_s - h_{TER}$$

where

R_i = initial target range, ft

A_{zi} = initial target azimuth, deg; positive right

ψ_i = aircraft heading at time of target designation, deg

Δ_x = translational motion of sensor in N-S direction after target designation, ft; positive north

Δ_y = translational motion of sensor in E-W direction after target designation, ft; positive east

z_s = simulated sensor height above terrain board reference, ft

h_{TER} = mean simulated height of terrain above terrain board reference, ft

Therefore, transforming to a body axis system with origin at the sensor location:

$$\begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ \sin \phi \sin \theta \cos \psi & \sin \phi \sin \theta \sin \psi & \sin \phi \cos \theta \\ -\cos \phi \sin \psi & +\cos \phi \cos \psi & \\ \cos \phi \sin \theta \cos \psi & \cos \phi \sin \theta \sin \psi & \cos \phi \cos \theta \\ +\sin \phi \sin \psi & -\sin \phi \cos \psi & \end{bmatrix} \begin{bmatrix} x_{TP} \\ y_{TP} \\ z_{TP} \end{bmatrix}$$

Summary of Equations

Orientation- The following parameters are used to drive the moving symbols which provide information on aircraft orientation:

Symbol	Parameter
AIRCRAFT HEADING	PSI
HORIZON LINE	THET, PHI

Situation- Aircraft position and velocity information in the horizontal and vertical planes are provided to the pilot through the following symbols:

Symbol	Parameter
HORIZONTAL	VELOCITY VECTOR
	XDH, YDH (TRANSITION) XDHAT, YDHAT (HOVER/BOB-UP)
	LONGITUDINAL AIRSPEED
	UBKTS
	HOVER POSITION
	EXH, EYH (BOB-UP)
VERTICAL	RADAR ALTITUDE
	HAGL
	RATE OF CLIMB
	ALTD

The velocity vector symbol is driven in the transition mode by the true values of ground velocity, XDH and YDH, and in the hover and bob-up modes by estimates of these values, XDHAT and YDHAT, respectively. These estimates are derived by complementary filtering techniques described in section 4 (Flight Control Display Logic). The ability to vary the scaling of the velocity vector is retained in the display dynamics program. Thus:

$$VVECX = \begin{cases} UKDXD * XDH & (\text{TRANSITION}) \\ UKDXD * XDHAT & (\text{HOVER/BOB-UP}) \end{cases}$$

and

$$VVECY = \begin{cases} UKDYD * YDH & (\text{TRANSITION}) \\ UKDYD * YDHAT & (\text{HOVER/BOB-UP}) \end{cases}$$

where UKDXD and UKDYD are constants the values of which may be selected by the researcher and which, in general, vary as a function of display mode.

In the bob-up mode, the hover position symbol moves in response to the variables EXH and EYH. Thus:

$$HOVX = UKDX * EXH$$

and

$$HOVY = UKDY * EYH$$

where UKDX and UKDY are constants whose values may be selected by the researcher.

Additional status information includes engine torque and lateral acceleration. The expression for engine torque is derived in section 4 (Torque Equation). The torque response to collective pitch is lagged by a first-order filter with a 0.1-sec time constant. Thus:

$$TRQ = TRQTRM + (THET\emptyset - TRQCB) * TRQG * \frac{10}{s + 10}$$

where

TRQTRM = trim value of percent torque in hover

TRQCB = trim value of collective pitch in hover

TRQG = percent torque change per degree of collective pitch

Lateral acceleration is driven by the parameter AYP.

Command- The cyclic director symbol provides "command" information in the horizontal plane which, if properly designed, allows the pilot to reach and maintain a stable hover. Thus,

$$VTIPX = VVECX + UKDTHT * THET * \left(\frac{T_1 s}{T_1 s + 1} \right)$$

$$VTIPY = VVECY + UKDPHI * PHI * \left(\frac{T_2 s}{T_2 s + 1} \right)$$

where UKDTHT and UKDPHI are constants the values of which may be selected by the researcher and which, in general, vary as a function of display mode; the nominal values of T_1 and T_2 are functions of display mode as follows:

	Transition	Hover/bob-up
T_1 , sec	50	10
T_2 , sec	10 for $PHIR \leq 0.1$ ∞ for $PHIR > 0.1$	10

In addition, a command heading symbol is provided; this symbol is driven by the difference between the current heading and the heading that existed at the time the bob-up display mode was selected (EPSIBU).

Finally, logic for a collective stick director is provided. The director logic is implemented as a weighted sum of altitude and altitude rate which drives the original rate of climb symbol; thus,

$$\text{ALTDRC} + \text{UKDALTD} * \text{ALTD} + \text{UKDHAGL} * (\text{HAGL} - 100)$$

For rate of climb information only, UKDHAGL is set to zero.

Fire control- The equations derived in section 4 (Fire Control Display Logic) for target azimuth and elevation angles, TADSX and TADSY, respectively, are implemented as

$$\text{XTP} = \text{RGT} * \cos(\text{AZAS} * \text{D2R}) + \text{DELX}$$

$$\text{YTP} = \text{RGT} * \sin(\text{AZAS} * \text{D2R}) - \text{DELY}$$

$$\text{ZTP} = \text{HPR} - \text{HTER}$$

where

$$\text{AZAS} = \text{AZA} + \text{PSIGT}$$

$$\text{DELX} = \text{YPR} - \text{YPR0}$$

$$\text{DELY} = \text{XPR} - \text{XPR0}$$

and RGT and AZA are constants selected by the researcher to represent initial target range and azimuth, respectively.

$$\text{XT} = \text{T11} * \text{XTP} + \text{T12} * \text{YTP} + \text{T13} * \text{ZTP}$$

$$\text{YT} = \text{T21} * \text{XTP} + \text{T22} * \text{YTP} + \text{T23} * \text{ZTP}$$

$$\text{ZT} = \text{T31} * \text{XTP} + \text{T32} * \text{YTP} + \text{T33} * \text{ZTP}$$

$$\text{TADSX} = \text{R2D} * \text{ATAN2}(\text{YT}, \text{XT})$$

$$\text{TADSY} = -\text{R2D} * \text{ASIN}(\text{ZT} / \text{SLANTR})$$

where $\text{SLANTR} = \sqrt{\text{XT}^2 + \text{YT}^2 + \text{ZT}^2}$.

5. WIND AND TURBULENCE

General

As described in reference 4, a model of atmospheric turbulence (BWIND) based on MIL-F-8785B (ref. 10) is implemented in the standard simulation software. Inputs required for the model are airspeed (VRW), the scale lengths (UAL, VAL, WAL), and the turbulence intensities (UDISP, VDISP, WDISP). Outputs

of the standard program are linear and angular turbulence velocities in body axes (UTURB, VTURB, WTURB, PTURB, QTURB, RTURB). A suggested revision to MIL-F-8785B (ref. 11) contains a low-altitude turbulence model; the changes basically involve:

1. Different relationships between scale lengths and altitude
2. Different relationship of turbulence intensities to mean wind speed and altitude
3. Linear turbulence velocities in wind axes rather than body axes

Approximations to these changes are implemented in the turbulence model for the advanced helicopter.

In addition a model of the steady wind, which includes provisions for wind shear in magnitude and direction, is included in the program.

Steady Wind

To provide the effects of steady wind and wind shear, the magnitude of the steady wind (V_w) is specified at two altitudes: 20 ft and 200 ft, AGL. Linear interpolation is used to determine mean wind speed between these altitudes. Beyond these altitude extremes the mean wind speed remains constant. Wind direction may be specified as a function of altitude in a similar fashion. The components of the steady wind in an earth-axis system are then derived for use as inputs to the standard simulation software.

Turbulence

The scale lengths required as inputs to BWIND are approximations to those presented in reference 11, that is,

$$L_w = \begin{cases} h & \text{for } h \geq 20 \text{ ft} \\ 20 & \text{for } h < 20 \text{ ft} \end{cases}$$

and

$$L_u = L_v = \begin{cases} 5h & \text{for } 200 \text{ ft} \geq h \geq 20 \text{ ft} \\ 100 & \text{for } h < 20 \text{ ft} \\ 1000 & \text{for } h > 200 \text{ ft} \end{cases}$$

To provide a valid turbulence model when airspeed is zero, steady wind speed (V_w) is used as the minimum value for the required airspeed input.

Finally, the vertical turbulence intensity is specified in reference 11 as being 10% of the mean wind speed measured at 20 ft, AGL. The ratio of the

horizontal turbulence intensities to the vertical intensity varies as a function of altitude from a value of 1.0 at 1,000 ft to 2.0 at zero altitude. A value of 2.0 is selected as being appropriate for the purposes of any simulations conducted using this model.

The outputs of BWIND are normally the six turbulence velocities in body axes. Two revisions to the output velocities are made for this model:

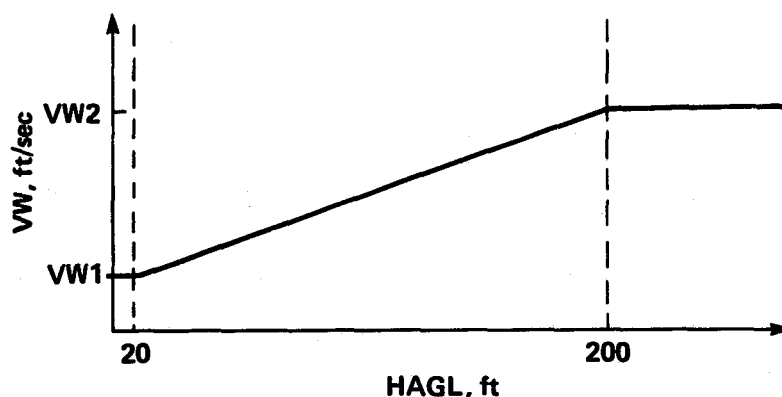
1. The angular turbulence velocity components are set to zero.
2. The linear turbulence velocity components are assumed to be in wind axes and are then transformed to earth axes and summed with the steady wind components.

In the frequency domain, the high-frequency cutoff of the turbulence components occurs in general at a frequency of V/L , according to reference 10. As an example, assume the mean wind speed to be constant at 15 knots and allow the altitude to vary from 20 to 200 ft AGL. The following variations in turbulence break frequencies result for hovering flight:

Altitude	V_w/L_w , rad/sec	$V_w/L_u = V_w/L_v$, rad/sec
20 ft	1.27	0.25
200 ft	0.13	0.025

Summary of Equations

Steady wind- The magnitude of the steady wind is specified at two values of HAGL; linear interpolation is used to determine wind speed between these values of altitude, that is,



Wind direction (PSIW) is then specified as a constant or a function of altitude. The components of the steady wind in earth axes are then calculated:

$$VNW = -VW \cdot \cos(\text{PSIW})$$

$$VEW = -VW * \sin(\text{PSIW})$$

$$VDW = 0$$

Turbulence- The scale lengths required as inputs by BWIND are specified as

$$WAL = \begin{cases} HAGL & \text{for } HAGL \geq 20 \\ 20 & \text{for } HAGL < 20 \end{cases}$$

$$UAL = VAL = \begin{cases} 5 * HAGL & \text{for } 200 \geq HAGL \geq 20 \\ 100 & \text{for } HAGL < 20 \\ 1000 & \text{for } HAGL > 200 \end{cases}$$

The minimum value of the BWIND input velocity VRW is set equal to the steady wind speed VW and the turbulence intensities are specified as follows:

$$WDISP = 0.1 * VWI$$

$$UDISP = VDISP = 0.2 * VWI$$

The three turbulence velocity components that are outputs of the program are transformed from wind axes to earth axes as:

$$VNTURB = -UTURB * \cos(\text{PSIW}) + VTURB * \sin(\text{PSIW})$$

$$VETURB = -UTURB * \sin(\text{PSIW}) - VTURB * \cos(\text{PSIW})$$

$$VDTURB = WTURB$$

Finally the earth-axis components of the steady wind are summed with these turbulence velocity components, yielding,

$$VTWN = VNW + VNTURB$$

$$VTWE = VEW + VETURB$$

$$VDW = VDTURB$$

6. CONCLUDING REMARKS

This report has described the derivation of a simplified mathematical model of a helicopter suitable for use in piloted simulator experiments involving tasks that may include relatively large variations in airspeed. The model was designed for a particular program of research which investigated the effects of control system and display variations for an attack helicopter mission; however, with appropriate modifications to the trim, stability, and

control parameters, it may also be used for the simulation of other types of helicopters performing different tasks. The major limitations of the model are: (1) only six degrees of freedom are simulated — no engine or rotor dynamics is included; and (2) for a given airspeed, the aerodynamic forces and moments are expressed as linear approximations of the actual forces and moments.

APPENDIX

AERODYNAMIC PARAMETERS

The appendix contains a summary of the parameters which characterize the aerodynamic forces and moments of the simulated helicopter. Table A-1 contains the mass and geometry constants required as inputs to the program. Table A-2 presents the reference values of the aircraft state and control parameters for level flight as a function of airspeed. Table A-3 contains the nonzero values of the aerodynamic forces and moments along the reference trajectory. Tables A-4 through A-9 present the values of the model's stability and control parameters as a function of airspeed.

TABLE A1.- MASS AND GEOMETRY CONSTANTS

Programming symbol	Engineering symbol	Definition	Units	Nominal value
XIXX	I_{xx}	{ Body axis moments of inertia	slug-ft ²	5,140
XIYY	I_{yy}			31,850
XIZZ	I_{zz}			30,850
XIXZ	I_{xz}	Cross-product of inertia	slug-ft ²	1,260
XMASS	m	Aircraft mass	slugs	453.42
XP		{ Pilot's design eye Position in body Axis coordinates	ft	5.12
YP				.0
ZP				-2.39
XPNVS		{ Sensor location in body axis coordinates	ft	15.5
YPNVS				.82
ZPNVS				.0

TABLE A2.- REFERENCE TRAJECTORY PARAMETERS

Programming symbol	Engineering symbol	UBKTS						Units
		-40	0	40	80	120	160	
WR	w_R	0.0	0.0	6.66	10.8	4.46	-1.89	ft/sec
A1SR	A_{1s_R}	-.16	-.16	-.44	-.61	-.90	-1.19	deg
B1SR	B_{1s_R}	-.45	-.45	2.97	4.86	7.17	9.47	deg
THETØR	θ_{OR}	15.75	15.75	14.03	13.23	15.09	16.95	deg
THETTRR	θ_{TRR}	21.46	21.46	13.44	8.69	8.02	7.35	deg

TABLE A3.- REFERENCE VALUES OF FORCES AND MOMENTS

Programming symbol	Engineering symbol	UBKTS											Units
		-40	-20	0	20	40	60	80	100	120	140	160	
XREF	X _R	3.559	2.934	2.308	2.59	2.806	2.692	2.151	1.503	0.4095	-0.684	-2.222	ft/sec ²
YREF	Y _R	2.047	2.049	2.050	1.710	1.365	1.187	1.009	1.203	1.396	1.587	1.777	ft/sec ²
ZREF	Z _R	-30.74	-31.39	-32.05	-30.68	-30.68	-31.98	-31.27	-32.38	-33.08	-32.67	-31.49	ft/sec ²
UMREF	M _R	-.169	-.0845	.0	-.0473	-.0947	-.190	-.2842	-.2845	-.2685	-.2525	-.2203	rad/sec ²

TABLE A4.- X-FORCE STABILITY AND CONTROL PARAMETERS

		UBKTS						Units
Programming symbol	Engineering symbol	-40	0	40	80	120	160	
XQ	X_q	1.39	1.39	1.92	1.99	1.71	1.42	ft/sec ² /rad/sec
α	X_u	-.0185	-.0185	-.0112	-.0328	-.0453	-.0577	ft/sec ² /ft/sec
XW	X_w	.0279	.0279	.0477	.0614	.0654	.0693	ft/sec ² /ft/sec
XTH ⁰	X_{θ_0}	.3328	.3328	.3067	.3715	.3460	.3205	ft/sec ² /deg
XB1S	X_{B1s}	.6147	.6147	.5332	.3984	.2799	.1614	ft/sec ² /deg
UKPBX	X_p	-1.45	-1.45	-1.45	-1.45	-1.45	-1.45	ft/sec ² /rad/sec
UKRBX	X_r	-.37	-.37	-.37	-.37	-.37	-.37	ft/sec ² /rad/sec
UKVBX	X_v	.004	.004	.004	.004	.004	.004	ft/sec ² /ft/sec
UKDA1SX	X_{A1s}	-.03	-.03	-.03	-.03	-.03	-.03	ft/sec ² /deg
UKDTHTRX	$X_{\theta_{TR}}$.0	.0	.0	.0	.0	.0	ft/sec ² /deg

^aNot explicitly included in aerodynamics (see sec. 2).

TABLE A5.- Y-FORCE STABILITY AND CONTROL PARAMETERS

		UBKTS						Units
Programming symbol	Engineering symbol	-40	0	40	80	120	160	
YP	Y_p	-1.463	-1.463	-2.152	-2.327	-1.933	-1.538	ft/sec ² /rad/sec
YR	Y_r	.7159	.7159	1.632	1.929	2.397	2.865	ft/sec ² /rad/sec
YV	Y_v	-.06	-.06	-.1106	-.1526	-.1972	-.2417	ft/sec ² /ft/sec
YA1S	Y_{A1s}	.6226	.6226	.6045	.5697	.5633	.5568	ft/sec ² /deg
YTHTR	$Y_{\theta TR}$.2	.2	.2	.2	.2	.2	ft/sec ² /deg
UKQBY	Y_q	-1.46	-1.46	-1.46	-1.46	-1.46	-1.46	ft/sec ² /rad/sec
UKDWBY	Y_w	-.0016	-.0016	-.0016	-.0016	-.0016	-.0016	ft/sec ² /ft/sec
UKDB1SY	Y_{B1s}	.019	.019	.019	.019	.019	.019	ft/sec ² /deg
UKDTH0Y	Y_{θ_0}	-.0268	-.0268	-.0268	-.0268	-.0268	-.0268	ft/sec ² /deg

TABLE A6.- Z-FORCE STABILITY AND CONTROL PARAMETERS

Programming symbol	Engineering symbol	UBKTS						Units
		-40	0	40	80	120	160	
ZQ	Z_q	0.3137	0.3137	-0.5676	-1.743	-2.873	-4.003	ft/sec ² /rad/sec
a	Z_u	-.0194	-.0194	-.1103	-.0118	.0124	.0366	ft/sec ² /ft/sec
ZW	Z_w	-.30	-.30	-.5223	-.7232	-.8171	-.9110	ft/sec ² /ft/sec
ZTHØ	Z_{θ_o}	-4.562	-4.562	-4.660	-5.5	-6.109	-6.718	ft/sec ² /deg
ZB1S	Z_{B_1s}	.0596	.0596	.6827	1.499	2.342	3.184	ft/sec ² /deg
ZH	Z_h	.0	.47	.0	.0	.0	.0	ft/sec ² /ft
UKPBZ	Z_p	-.08	-.08	-.08	-.08	-.08	-.08	ft/sec ² /rad/sec
UKRBZ	Z_r	2.5	2.5	2.5	2.5	2.5	2.5	ft/sec ² /rad/sec
UKVBZ	Z_v	-.036	-.036	-.036	-.036	-.036	-.036	ft/sec ² /ft/sec
UKDA1SZ	Z_{A_1s}	.0	.0	.0	.0	.0	.0	ft/sec ² /deg
UKDTHTRZ	$Z_{\theta_{TR}}$.0	.0	.0	.0	.0	.0	ft/sec ² /deg

^aNot explicitly included in aerodynamics (see sec. 2.2).

TABLE A7.- L-MOMENT STABILITY AND CONTROL PARAMETERS

Programming symbol	Engineering symbol	UBKTS						Units
		-40	0	40	80	120	160	
ULP	L_p	-2.86	-2.86	-3.189	-3.246	-2.962	-2.678	rad/sec ² /rad/sec
ULR	L_r	.0292	.0292	.2712	.3472	.4960	.6448	rad/sec ² /rad/sec
ULV	L_v	-.0237	-.0237	-.0294	-.0246	-.0244	-.0242	rad/sec ² /ft/sec
ULA1S	L_{A1s}	.7676	.7676	.7588	.7322	.7335	.7348	rad/sec ² /deg
ULTTR	$L_{\theta TR}$.06	.06	.06	.06	.06	.06	rad/sec ² /deg
UKQBL	L_q	-1.2	-1.2	-1.2	-1.2	-1.2	-1.2	rad/sec ² /rad/sec
UKDWBL	L_w	-.001	-.001	-.001	-.001	-.001	-.001	rad/sec ² /ft/sec
UKDB1SL	L_{B1s}	.04	.04	.04	.04	.04	.04	rad/sec ² /deg
UKDTH0L	$L_{\theta 0}$	-.1	-.1	-.1	-.1	-.1	-.1	rad/sec ² /deg

TABLE A8.- M-MOMENT STABILITY AND CONTROL PARAMETERS

		UBKTS						
Programming symbol	Engineering symbol	-40	0	40	80	120	160	Units
UMQ	M_q	-0.5092	-0.5092	-0.5858	-0.9424	-1.113	-1.283	rad/sec ² /rad/sec
α	M_u	.0025	.0025	.00144	.00197	.00167	.00136	rad/sec ² /ft/sec
UMW	M_w	-.00013	-.00013	.00065	.00051	-.00199	-.00449	rad/sec ² /ft/sec
UMTHØ	M_{θ_0}	-.0014	-.0014	.0631	.0888	.1351	.1813	rad/sec ² /deg
UMB1S	M_{B_1S}	-.1242	-.1242	-.1277	-.1330	-.1495	-.1659	rad/sec ² /deg
UKPBM	M_p	.22	.22	.22	.22	.22	.22	rad/sec ² /rad/sec
UKRBM	M_r	.03	.03	.03	.03	.03	.03	rad/sec ² /rad/sec
UKVBM	M_v	.0001	.0001	.0001	.0001	.0001	.0001	rad/sec ² /ft/sec
UKDA1SM	M_{A_1S}	.008	.008	.008	.008	.008	.008	rad/sec ² /deg
UKDTHTRM	$M_{\theta TR}$	-.0022	-.0022	-.0022	-.0022	-.0022	-.0022	rad/sec ² /deg

^aNot explicitly included in aerodynamics (see sec. 2.2).

TABLE A9.- N-MOMENT STABILITY AND CONTROL PARAMETERS

UBKTS								
Programming symbol	Engineering symbol	-40	0	40	80	120	160	Units
UNP	N_p	-0.1883	-0.1883	-0.1359	-0.1236	-0.0767	-0.0298	rad/sec ² /rad/sec
UNR	N_r	-.4409	-.4409	-.8246	-.9426	-1.151	-1.355	rad/sec ² /rad/sec
UNV	N_v	.0033	.0033	.0201	.0209	.0232	.0255	rad/sec ² /ft/sec
UNTHØ	N_{θ_0}	.1618	.1618	.0988	.0726	.0907	.1088	rad/sec ² /deg
UNALS	N_{A_1s}	.013	.013	.0097	.0064	.0067	.0070	rad/sec ² /deg
UNTTR	$N_{\theta_{TR}}$	-.08	-.08	-.08	-.08	-.08	-.08	rad/sec ² /deg
UKQBN	N_q	-.15	-.15	-.15	-.15	-.15	-.15	rad/sec ² /rad/sec
UKDWB	N_w	.002	.002	.002	.002	.002	.002	rad/sec ² /ft/sec
UKDB1SN	N_{B_1s}	.003	.003	.003	.003	.003	.003	rad/sec ² /deg

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16. Abstract <p>This report documents a mathematical model of an advanced helicopter; the model is suitable for use in control/display research involving piloted simulation. The general design approach for the six-degree-of-freedom equations of motion is to use the full set of nonlinear gravitational and inertial terms of the equations and to express the aerodynamic forces and moments as the reference values and first-order terms of a Taylor series expansion about a reference trajectory defined as a function of longitudinal airspeed. Provisions for several different specific and generic flight control systems are included in the model. The logic required to drive various flight control and weapon delivery symbols on a pilot's electronic display is also provided. Finally, the model includes a simplified representation of low-altitude wind and turbulence effects. This model has been used in a piloted simulator investigation, recently conducted at Ames Research Center, of the effects of control system and display variations for an attack helicopter mission.</p>					
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